Uncoordinated MAC for Adaptive Multi-Beam Directional Networks: Analysis and Evaluation

Greg Kuperman
MIT Lincoln Laboratory
Lexington, MA, USA
gkuperman@ll.mit.edu

Robert Margolies
Columbia Universtiy
New York, NY, USA
rsm2156@columbia.edu

Nathaniel M. Jones
MIT Lincoln Laboratory
Lexington, MA, USA
njones@ll.mit.edu

Aradhana Narula-Tam
MIT Lincoln Laboratory
Lexington, MA, USA
arad@ll.mit.edu

***Abstract*—In this paper, we consider medium access control (MAC) policies for emerging systems that are equipped with fully digital antenna arrays which are capable of adaptive multi-beam directional communications. With this technology, a user can form multiple simultaneous transmit or receive beams, allowing for greater spatial reuse and higher network throughput. The enabling technology that we consider is the ability to use digital post-processing to form multiple receive beams in real-time without a priori knowledge of the time and angle-of-arrival of the transmission.**

We present a novel unslotted, uncoordinated ALOHA-like random access MAC policy for multi-beam directional systems that asymptotically achieves the capacity of the network. Such an approach is particularly useful for systems where propagation delay makes the overhead associated with any sort of coordination prohibitive. We also consider the impact of numerous practical considerations including power constraints, latency, and beamwidth on the performance of our MAC policy.

***Index Terms*—Medium Access Control, MAC, Adaptive Beamforming, Multibeam, Directional Networking, Random Access, Smart Antennas**

I. Introduction

Fully digital beamforming antenna arrays that are capable of adaptive multi-beam communications are quickly becoming a reality. These antenna arrays allow users to form multiple simultaneous transmit or receive beams within the same frequency channel, while adaptively steering nulls to minimize interference with other users. The benefits of such a directional beamforming system are numerous, including higher spatial reuse, higher data rates, and longer transmit ranges [1].

The enabling technology we consider is the ability of a fully digital antenna array to dynamically form multiple directional receive beams at the time of reception without prior knowledge of the time or angle-of-arrival (AoA) of each incoming transmission [2, 3]; i.e., the ability to form receive beams *a posteriori*. This a posteriori reception is accomplished by placing an analog-to-digital converter (ADC) behind each antenna element, which allows the incoming signal to be fully digitized and post-processed [4]. On the transmit side, multiple data streams are digitally superimposed to produce multiple independent transmit beams. This approach has high computational complexity [5], but has already been applied to radar systems [6], and development is accelerating for its use in mobile ad hoc communication systems [7].

This fully digital adaptive beamforming approach is in contrast to conventional beamforming systems where a receiver must schedule a beam in the direction of the transmitter for successful reception [8]. Conventional beamforming requires users to know when and where to point both the transmit and receive beams, necessitating complex coordination between all users of the network. With the ability to dynamically form receive beams in multiple directions on-the-fly, a user can now behave as an omni-directional receiver while maintaining the benefits of directionality. This enables the development of an uncoordinated random access medium access control (MAC) that was previously not possible for directional systems.

An uncoordinated random access MAC works well in systems with large propagation time relative to packet transmission time. There are numerous examples of such networks. For example, an emerging area for communications are airborne networks [9], where users can easily be hundreds of kilometers apart from one another, leading to propagation delays on the order of milliseconds. This includes both civilian [10] and military airborne communications [11]. For military networks, directional communications holds particular appeal due to its inherent anti-jam properties [12]. Propagation delay can be an issue in short-range networks as well. With the high gain that these antenna arrays can achieve, data rates of tens of gigabits per second will soon be realizable [13]. At 10 Gbps, a 1 Kb packet is transmitted in 0.1 μs, which is equivalent to the propagation delay across a distance of 30 meters. A random access MAC that does not make use of reservation messages and operates in an unslotted, uncoordinated fashion will be particularly useful for these types of networks.

In this paper, we develop an uncoordinated, unslotted ALOHA-like random access policy for adaptive multi-beam directional systems that we title Multi-Beam Uncoordinated Random Access MAC (MB-URAM). Our approach is uncoordinated in the sense that each node acts independently of one another. MB-URAM does not make use of any reservation messages (such as scheduling or RTS/CTS) and does not need to synchronize time slots or transmissions. Traditionally, in omni-directional systems, random access approaches have lower network throughput than TDMA scheduling [14], and

Distribution A: public release, unlimited distribution. This work is sponsored by the Assistant Secretary of Defense Research and Engineering via Air Force contract #FA8721-05-C-0002. Opinions, inter-pretations, conclusions and recommendations are those of the author and are not necessarily endorsed by the United States Government.

moving from a slotted to an unslotted scheme results in an additional decrease in network throughput [15]. However, for multi-beam directional systems, we demonstrate that MB-URAM asymptotically achieves the maximum possible throughput for any MAC approach, even a scheduled one.

In addition to presenting an asymptotic analysis of MB-URAM, we analyze MB-URAM with respect to realistic constraints that limit achievable throughput. We present analysis and modifications for MB-URAM operating under practical considerations, such as power constraints, realistic beamwidths, and latency requirements. For power constraints, we demonstrate that the problem of selecting the optimal set of beams such that power constraints are not violated is an NP-hard problem, but is optimally solvable in pseudo-polynomial time. Larger beamwidths cause increased interference between users, which decreases throughput; we present analysis showing how beamwidth affects throughput using MB-URAM, and we suggest modifications to increase the policy's performance. With respect to latency, MB-URAM is tunable and we demonstrate that most of the network capacity is achieved with relatively low delay in the network.

The remainder of the paper is organized as follows. In Section II, we examine previous work in the area. In Section III, we present the antenna and network model. In Section IV, we introduce MB-URAM, and demonstrate that it asymptotically achieves the network capacity. In Section V, we analyze MB-URAM under a variety of practical considerations.

II. Previous Work

Previous works for multi-beam systems have all considered approaches that require significant coordination between users. In [16] and [17], time division multiple access (TDMA) MAC policies are considered. TDMA schemes require synchronization and a complex distributed scheduling scheme to be run across all of the nodes in the network. Slotted ALOHA-like schemes are proposed in [18], but synchronizing time slots across the network is challenging and potentially leads to wasted throughput. Furthermore, any time slotted approach, whether it be TDMA or slotted ALOHA, does not work well for systems with large propagation delay relative to the packet size: large guard times must be included so that packets can be guaranteed to be received within a time slot and not interfere with receptions in adjacent time slots [15]. Asynchronous schemes based on 802.11 that include channel reservation messages (RTS/CTS) have been proposed in [19]. These schemes also do not work well when propagation delay is large relative to packet transmission time; the network spends most of its time waiting for the channel to be reserved as opposed to transmitting data.

We note that a large body of work exists for single-beam directional networks (where a node can form only one transmit or receive beam at a time), with a survey of MAC protocols for these systems being found in [20]. Due to the limiting nature of only being able to form a single beam a time, none of these consider an uncoordinated random access MAC approach.

III. Antenna and Network Model

We consider an array of antenna elements, where each element is equipped with an ADC. The antenna array is capable of digital beamforming, where the signals from each element are combined to form multiple independent beams that can be steered in all directions. By having an ADC at each element on the receive side, the digital version of the received signal can be copied and parallel-processed to effectively form directional receive beams in all possible angle-of-arrivals. Adaptive processing techniques allow nulls to be formed to further suppress interference from adjacent beams. To transmit, a complementary process is used to form a beam in the direction of the destination [1, 2].

For simplicity, we consider a network operating in a two-dimensional plane. We note that our results can be easily extended to three dimensions. We assume a linear antenna array composed of a total of K elements (for a 3-D network, we would use a square array). An array of K elements can form up to K simultaneous beams and nulls [1]. Each beam potentially carries independent information. The antenna array can transmit a total of P watts of power. The total transmit power P is fixed; hence, forming multiple transmit beams results in splitting the power between the multiple beams. If k transmit beams are formed and the i^{th} beam uses p_i watts, then the system power constraint is $\sum_{i=1}^{k} p_i \leq P$.

We consider a network that supports only a single data rate, and that the power required for any transmission to be successful between two users at that rate is known. Furthermore, we assume all packets in the network are of the same size. Hence, we normalize the data rate to be one packet per time unit. In Section V, we extend our approach to allow for different rates across any particular link by varying the transmit power. For initial analysis, we assume that the system is single-channel. In Section V, we consider a multi-channel system.

If a node is not transmitting, we say that it is in "receive mode" and is listening across all possible receive beams. If a packet being received at node v overlaps for any fraction of time with node v's transmission, the received packet at v is assumed to be fully lost.

Beamwidth is a function of both the number of elements and the choice of beamforming algorithm. Without any adaptive techniques, an array of K elements at half-wavelength spacing will have a half-power beamwidth of approximately $\frac{100^\circ}{K}$ [1]. Due to the variety of potential beam shapes from various beamforming approaches, as well as to keep our analysis tractable, we assume a "flat-top" antenna model, which ignores side lobes and treats a directional beam as a wedge of width θ degrees [21]. We assume any transmission or reception outside of the beam has zero gain and can be ignored. Using this model for directional beamforming, we have the following constraints on reception and transmission: (1) a node can receive simultaneous directional beams that are separated by at least $\frac{1}{2}\theta$ degrees, (2) a node cannot transmit simultaneously to multiple users that are spatially separated by less than $\frac{1}{2}\theta$ degrees, and (3) a node cannot both transmit and receive simultaneously (half-duplex constraints).

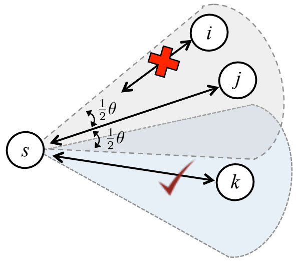


Fig. 1: Flat-top antenna model from the perspective of user s

In Figure 1, this flat-top model is shown from the perspective of user s. If all users have a beamwidth of θ, then while user s has an active beam with j, s *cannot* have a beam to or from i. However, s *can* have a beam to or from k.

For our analysis, we consider a network of N users where all users are within communication range of one another; i.e., a complete graph. We assume any particular user knows the location of all of its neighbors, which means that each user knows the power necessary to transmit to any other user. Users' locations may be learned either through side channel information, or through a neighbor discovery process that will be discussed in Section V. We note that neighbor discovery is a requirement for both coordinated and uncoordinated schemes. Due to the uncoordinated nature of our proposed MAC, our work can be easily extended to multi-hop networks.

A key metric that we use throughout the paper is *network throuhgput*. We define network throughput as the number of successful packet transmissions per time unit across the network of N users. This metric is also sometimes referred to as the network *goodput* [22]. Additionally, we will often refer to the *network capacity*, which represents the maximum possible network throughput. For analytical purposes, we assume that all users always have data to send to any other users. In Section V, we relax this assumption and consider the effect of stochastic arrivals on network performance.

IV. Uncoordinated Random Access MAC for Multi-Beam Directional Networks

In this section, we develop Multi-Beam Uncoordinated Random Access MAC (MB-URAM). In Section IV-A, we analyze the network capacity under idealized settings. In Section IV-B, we develop MB-URAM and demonstrate that it asymptotically achieves the network capacity.

Our goal in this section is to perform asymptotic analysis of the network throughput for multi-beam channel access schemes . Thereby, we assume that beamwidths approach zero and that sufficient power exists to support any desired number of simultaneous transmissions. In Section V, we consider the performance of MB-URAM under practical constraints such as wider beamwidths and power limitations. We note that the assumptions of narrow beamwidth and sufficient power are not entirely unrealistic. As the number of array elements grows large and more sophisticated adaptive beamforming algorithms are used, almost arbitrarily small beamwidths can be realized [1]. Furthermore, a large numbers of antenna elements results in high antenna gain, which allows for sufficient power to form many transmit beams.

A. Multi-Beam Network Throughput Analysis

We begin by deriving the network capacity for any possible half-duplex MAC policy. We assume we have a network of N users, all within range of each another. Hence, there exists $N(N-1)$ possible links. Any connection between two users supports a rate of one packet per time unit. With beamwidths approaching zero, there is no interference between adjacent beams; on the receive side, all incoming packets can be successfully resolved. With the assumption of sufficient power to support a link to any desired number of users, a user can simultaneously transmit to all other users if it chooses to do so. We assume half-duplex constraints; hence, only self-interference exists in our network. For a packet transmission to be successful, the receiving node cannot transmit to any other node during the duration of a packet reception.

Recall that network throughput refers to the total successful packet transmissions per time unit across all links throughout the network. If during a unit of time, K links successfully transmitted a packet, then the achieved network capacity is K. The network capacity is the maximum network throughput that can possibly be supported.

Theorem 1. *The network capacity for a multi-beam network with half-duplex constraints is* $\frac{1}{4}N^2$.

Proof: At any given moment, a user can either be transmitting or receiving. We split the users into two groups: n transmitters and $N-n$ receivers. Each transmitter sends a packet to each receiver. The total number of active links is $n(N-n)$. Maximizing with respect to n, we find that the optimal solution is to have $\frac{1}{2}N$ transmitters and $\frac{1}{2}N$ receivers, which gives a network capacity of $\frac{1}{4}N^2$. ■

A similar upper bound on network capacity for multi-beam systems was demonstrated in [17]. When N is even, we can evenly split the total users between the transmit and receive group, resulting in the network capacity of $\frac{1}{4}N^2$. When N is odd, we split the users into $\frac{N+1}{2}$ transmitters and $\frac{N-1}{2}$ receivers, which results in a network capacity slightly lower than, but still upper bounded by, $\frac{1}{4}N^2$. For the remainder of this paper, we assume an even number of users.

The network capacity of $\frac{1}{4}N^2$ is simply the maximum number of links that can be active at any given time; it does not specify a feasible schedule, nor is this rate necessarily achievable. If we assume some sort of centralized scheduling agent and we ignore propagation delay, guard times, and overhead associated with synchronization, a simple policy that proportionally serves all links and asymptotically achieves the upper bound is the following: in each time slot, randomly select $\frac{N}{2}$ users as transmitters and $\frac{N}{2}$ as receivers, where each transmitter simultaneously sends a packet to all $\frac{N}{2}$ receivers. Consider two users, v_1 and v_2. The probability in any given time slot that v_1 is a transmitter and v_2 is a receiver is $\frac{1}{4}$. Hence, as time goes to infinity, every link is active in $\frac{1}{4}$ of all time slots. There are $N(N-1)$ links in the network, with each operating at $\frac{1}{4}$ utilization. This gives a network throughput of $\frac{1}{4}N(N-1)$, which approaches $\frac{1}{4}N^2$ as N grows large. We call this policy Multi-Beam Synchronous Scheduling (MB-SS).

B. Random Access MAC for Multi-Beam Networks

We now develop a random access MAC policy that asymptotically achieves the network capacity for multi-beam systems. First, we define the model and notation that we use to describe and evaluate a random access MAC. Next, we present a synchronous time slotted random access MAC that achieves the network capacity. We then consider an asynchronous, uncoordinated system, and we develop a random access MAC that asymptotically achieves the network capacity.

1) Random Access MAC Model and Notation: To send a packet, a node will wait a random amount of time R, after which it will enter transmit mode, where that node will send one packet to each of its neighbors. After transmitting the packet, the node again waits a random amount of time R before sending the next packet. During the waiting time, a node is in receive mode and can successfully receive a packet from any of its neighbors if it arrives entirely within the receive window (i.e., it does not overlap with the start or end of a transmit period). After the wait time R expires, a node will transmit regardless of whether or not it is currently receiving a packet. We refer to a node that is not transmitting and is able to receive as *idle*. The probability density function of the idle time R is $f_R(r)$, and has expected value $\mathbb{E}[R]$.

We define an *interval* as the amount of time between two consecutive packet transmission starts. Since we consider a single rate system where transmit rates have been normalized, a packet takes one unit of time to transmit. All of the policies we consider have a single packet transmission followed by some idle period. Hence, an interval has length $1 + R$ with expected duration $1 + \mathbb{E}[R]$.

Each node in the network operates independently and in uncoordinated fashion with respect to the behavior of any other nodes. Furthermore, nodes do not change their behavior based on whether or not they are currently receiving a packet. The probability that a packet arrives at an idle node is simply the time average that the destination node is idle:

$$P_{idle} = \frac{\mathbb{E}[R]}{1 + \mathbb{E}[R]} \tag{1}$$

If a packet arrives during an idle period, the receiver must remain idle for the duration of the packet reception for the transmission to be successful. The probability distribution function for the amount of time remaining in a idle period starting at time t is labeled $Y(t)$, which is known as the forward recurrence time [23]. Given that a packet arrival starts during an idle period, we label P_Y as the probability that the node stays idle for at least one unit of time:

$$P_Y = \mathbf{P}(Y(t) \geq 1) \tag{2}$$

The probability that a packet is successfully received is the probability that a packet arrives during an idle period and that the remaining time in the idle period is at least one unit:

$$P_{recv} = P_{idle} P_Y \tag{3}$$

The transmission rate for each link is one packet per interval. We label the time average transmit rate as:

$$\lambda = \frac{1}{1 + \mathbb{E}[R]} \tag{4}$$

Since each node acts independently and without coordination, we consider each packet transmission independently. The successful throughput T per link is the packet transmit rate λ multiplied by the probability of success of any individual packet P_{recv}:

$$T = \lambda P_{recv} = \frac{1}{1 + \mathbb{E}[R]} \frac{\mathbb{E}[R]}{1 + \mathbb{E}[R]} \mathbf{P}(Y(t) \geq 1) \tag{5}$$

The network throughput is then the number of links in the network multiplied by link throughput: $T \cdot N(N-1)$, which goes to TN^2 as N becomes large.

2) Random Access MAC for a Synchronous Slotted System: We begin by analyzing a synchronous time slotted system, for which we propose a simple random access policy that achieves the same link utilization for each link as MB-SS. In each time slot, a user randomly and independently selects being either a transmitter or a receiver with a probability of $\frac{1}{2}$. If a user is a transmitter, it sends packets to all other $N-1$ users in the network. If a user is a receiver, it listens for packets from all potential transmitters. We call this approach Multi-Beam Synchronous Random Access (MB-SRA).

We now show that this approach achieves the network capacity as the number of users grows large. We note that since the system is perfectly synchronized across the network, a packet always arrives perfectly at the beginning of a time slot. Hence, if a packet arrives at an idle node, then the packet will be successfully received. In other words, $P_Y = 1$.

Lemma 1. *Multi-Beam Synchronous Random Access achieves the network throughput of $\frac{1}{4}N(N-1)$. As N grows large, the throuhgput approaches the network capacity of $\frac{1}{4}N^2$.*

Proof: The transmit rate of any node is $\lambda = \frac{1}{1+\mathbb{E}[R]} = \frac{1}{2}$, and the probability that a packet sees an idle node is $P_{idle} = \frac{\mathbb{E}[R]}{1+\mathbb{E}[R]} = \frac{1}{2}$. Since we assume the system is perfectly synchronized, $P_Y = 1$. Hence, the achieved link throughput is $T_{MB-SRA} = \lambda P_{idle} P_Y = \frac{1}{2} \cdot \frac{1}{2} \cdot 1 = \frac{1}{4}$.

With $N(N-1)$ links in the network, the achieved network throughput is $\frac{1}{4}N(N-1)$. As N grows large, the network throughput approaches $\frac{1}{4}N^2$. ∎

For this scheme to achieve network capacity, we ignored any overhead associated with synchronization. A perfectly time-slotted system is not practical in networks of interest where propagation delay is long relative to packet duration.

3) Random Access MAC for an Asynchronous System: We now consider an asynchronous network without coordinated time slots. Since propagation delay can be arbitrarily large with respect to message size, carrier sense multiple access (CSMA) can cause inaccurate channel assessment and the RTS/CTS message exchange can become prohibitive [15]. We develop a policy where each node acts independently and in an uncoordinated fashion.

As a baseline, we consider MB-SRA, but now operating asynchronously. We refer to this scheme as Multi-Beam Asynchronous Random Access (MB-ARA). A node still operates using packet length time slots, as described above. These time slots are not necessarily aligned with any other user's time slots. We maintain a transmit rate at any node of $\lambda = \frac{1}{2}$; hence, the probability that a packet arrives at node that is idle is $P_{idle} = \frac{1}{2}$. We assume no guard times; i.e., a packet occupies an entire time slot. Since the system is asynchronous and propagation time adds additional delay to packets, the probability that time slots are perfectly aligned across any pair of users is effectively zero; in other words, we no longer assume $P_Y = 1$. Since a packet will never arrive at the beginning of a time slot, a receiving node must be idle in the following time slot for a packet to be successfully received. Hence, $P_Y = P_{idle}$.

The link throughput of this policy is: $T_{MB-ARA} = \lambda P_{idle} P_Y = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$, which gives a network throughput of $\frac{1}{8}N^2$ as N grows large. This is exactly half of the throughput of MB-SRA, which is as one would expect: the classic ALOHA result for omni-directional systems has a network throughput decrease from $\frac{1}{e}$ to $\frac{1}{2e}$ when going from slotted to unslotted [15].

To provide insight, we further optimize MB-ARA. Maximizing throughput T_{MB-ARA} with respect to P_{idle} (where $\lambda = 1 - P_{idle}$), we obtain a value of $P_{idle} = \frac{1}{3}$, which gives a link throughput of $\frac{4}{27} = 14.81\%$. Instead of being idle half of the time and transmitting half of the time, a node is idle $\frac{2}{3}$ of the time, and transmitting $\frac{1}{3}$. A packet arriving at an idle node has a higher probability of being successfully received, but, this higher probability of reception success comes at the cost of a lower transmit rate.

For a packet to have a high probability of being successfully received, a receiver should remain idle for as long as possible. Furthermore, any idle period that is a packet length or shorter is not useful; no packet can ever be received during that idle period, and that time would have been better utilized transmitting. For idle time R, we wish to have a distribution that allows P_Y to approach 1, and also prevents any wasted transmission time due to short idle periods.

We now define the capacity achieving distribution R_C, where x is a tunable parameter:

$$f_{R_C}(r) = \begin{cases} r = 0, & \frac{x-1}{x} \\ r = x, & \frac{1}{x} \end{cases} \tag{6}$$

If we maintain the requirement that a node operates using packet length time slots, then x can be set to an integer value, but x does not necessarily have to be integer; the only requirement is that $x > 1$. If $x \leq 1$, then no packet can be successfully received during an idle time. As x grows large, with high probability there is an idle time of zero, and with low probability, an idle time of x. The expected value of R_C is:

$$\mathbb{E}[R_C] = 0 \cdot \frac{x-1}{x} + x \cdot \frac{1}{x} = 1 \tag{7}$$

The probability of a node being idle $P_{idle,C}$ and the transmission rate per link λ_C for all values of x are the following:

$$P_{idle,C} = \frac{\mathbb{E}[R_C]}{1+\mathbb{E}[R_C]} = \frac{1}{2} \tag{8}$$

$$\lambda_C = \frac{1}{1+\mathbb{E}[R_C]} = \frac{1}{2} \tag{9}$$

We next consider the probability that a packet is successfully received given that it arrives at an idle node. Using the capacity achieving distribution, a node is idle for x units of time. For a unit-length packet to be received successfully, it must have arrived during the first $x-1$ units of time of an idle period. Since all transmissions are uncoordinated, a packet has equal chance of arriving during any moment of that idle period. The probability that that node will remain idle for the packet duration using the capacity achieving distribution is:

$$P_{Y_C} = \mathbf{P}(Y_C(t) > 1) = \frac{x-1}{x} \tag{10}$$

As x goes to infinity, P_{Y_C} goes to 1. The Multi-Beam Uncoordinated Random Access MAC (MB-URAM) has the following simple policy: transmit a packet of unit length to all neighbors, and then go idle for R_C time.

Theorem 2. *Multi-Beam Uncoordinated Random Access MAC asymptotically achieves a network throughput upperbound of $\frac{1}{4}N^2$ as x goes to infinity, beamwidth goes to zero, and the number of users N grows large.*

Proof: As $x \to \infty$, $\mathbb{E}[R_C]$ stays constant at 1, and P_{Y_C} approaches 1. Hence, as $x \to \infty$, the per link throughput is:

$$\lim_{x\to\infty} T_{\substack{MB\\URAM}} = \frac{1}{1+\mathbb{E}[R_C]} \frac{\mathbb{E}[R_C]}{1+\mathbb{E}[R_C]} P_{Y_C} = \frac{1}{2} \cdot \frac{1}{2} \cdot 1 = \frac{1}{4}$$

As $x \to \infty$, each link has a utilization of $\frac{1}{4}$. With $N(N-1)$ links in the network, the achieved network throughput is $\frac{1}{4}N(N-1)$, which approaches $\frac{1}{4}N^2$ as N grows large. ■

We make a few observation regarding MB-URAM. While on average, a node is transmitting 50% of the time, with large values of x, any particular link will see long transmit periods and long receive periods. The consequence of such a scheme is that long delays can be potentially incurred by a packet waiting to be transmitted. We discuss practical latency requirements in more detail in Section V.

Finally, we note that the uncoordinated random access MAC approach outlined above has an expected packet failure rate of 50%. By being able to characterize the expected loss, we can develop techniques to overcome this loss. While a more thorough analysis of such mitigation approaches will be the subject of future work, some potential approaches are forward error correction (FEC) codes across the packets (such as fountain coding [24]), selective or negative acknowledgments (ACKs), or a combination of the FEC and ACKs.

In the next section, we consider the performance impact of practical limitations on our uncoordinated multi-beam random access MAC (including more realistic assumptions on power, beamwidth, and latency constraints).

V. Practical Considerations

In the previous section, we presented an uncoordinated random access MAC approach for multi-beam directional systems that asymptotically achieves the network capacity. For this analysis, we made the following assumptions: (1) sufficient power to support any number of simultaneous beams, (2) beamwidths are sufficiently narrow that interference between users can be ignored, (3) that the inter-arrival time between packet transmissions can grow arbitrarily large, and (4) that the location of each user is known in advance. While system design is trending towards being able to support a very high number of simultaneous transmit and receive beams with almost arbitrarily narrow beamwidths, we now evaluate the proposed MAC without the idealized assumptions.

In Section V-A, we consider more realistic power constraints at the transmitter for single and multi-rate systems. In Section V-B, we analyze the latency characteristics of our uncoordinated random access approach when we no longer allow the idle time to have infinite variance. In Section V-C, we remove the assumption of arbitrarily narrow beamwidths, and consider a system where transmissions and receptions can interfere with one another. In Section V-D, we discuss neighbor discovery for multi-beam directional networks.

A. Multi-Beam Random Access under Power Constraints

In the ideal version of the uncoordinated multi-beam random access policy presented in Section IV, when a node enters a transmit phase, a packet is transmitted to all users and all queues are serviced at the same rate. When only a subset of possible beams can be formed, queues may be serviced at different rates and backlogs can grow. Here we constrain the maximum total transmission power of an array to be P, where power must be portioned across the set of active beams; some beams might receive no power. Let B be the set of all possible beams to other users, and let $p_i \geq 0$ be the power allocated across beam i to transmit to neighbor i. The array power constraint is then $\sum_{i \in B} p_i \leq P$. We wish to find a power allocation that maximizes throughput. We first consider a system with a single fixed data rate, and then we extend our approach to a more general system where different transmit power results in a different transmission rate.

Each node operates in an uncoordinated fashion, and thus makes decisions independently. We study each transmitter individually, and solve this as a multi-user downlink problem. For each transmission we apply the Max-Weight policy for power allocation [25]. This policy does not require knowledge of the arrival rate vector, and is known to be throughput optimal in that it supports all arrival rates that can be supported by any policy. For each link i, the policy chooses a utility $c_i = r_i q_i$, where r_i is the rate of link i and q_i is the queue backlog of packets to be sent across link i. The Max-Weight policy then chooses to activate the set of beams $\arg\max \sum_{i \in B} c_i$ subject to the total power constraint.

1) Single Rate: For each link i, we assume the power p_i to support the rate of one packet per unit time is known. Since the system supports only a single rate, $r_i = 1, \forall i \in B$; hence $c_i = q_i$. Let b_i be a decision variable where $b_i = 1$ if a beam to user i is active, and 0 otherwise. The optimization problem can be written as $\max_{b_i} \sum_i c_i b_i$ s.t. $\sum_i p_i b_i \leq P$ and $b_i \in \{0, 1\}, \forall i$. This is a 0-1 knapsack problem where c_i is the value of item i, p_i is the weight of item i, and P is the total knapsack capacity. This problem is known to be NP-Hard, and can be solved via a pseudo-polynomial dynamic programming algorithm with run time $\mathcal{O}(|B| \cdot P)$ [26]. The algorithm is said to run in pseudo-polynomial time because its runtime is dependent on the number of bits needed to represent the maximum transmit power. If the number of bits needed to represent P is polynomial bounded, then the algorithm runs in polynomial time.

2) Continuous Rates: Instead of assuming the system can only support a single rate, we now allow for any possible rate to neighbor i by varying the transmit power p_i. We use an SNR model to calculate the rate of each link i as $r_i = W \log_2(1 + p_i s_i)$, where $p_i s_i$ is the SNR of link i at transmit power p_i. Parameters s_i are assumed to be known for all links. Let the bandwidth scalar $W = 1$ for simplicity. The optimization problem can be written as $\max_{p_i} \sum_i q_i \log_2(1 + p_i s_i)$ s.t. $\sum_i p_i \leq P$ and $p_i \geq 0, \forall i$. This is a weighted power allocation problem, where queue size q_i is the weight of link i; this can be solved in $\mathcal{O}(|B|^2)$ time with a water-filling solution [27]. We note that for variable rates, MB-URAM can use packets of equal time duration; all packets in the network, regardless of rate, will be the same length in time. This way, when a user transmits to multiple receivers, no transmission will finish before any other.

B. Latency Constraints

We consider two users: transmitter u and receiver v. When a packet from u arrives at an idle user v, v must stay idle for at least a packet in length for u's transmission to v to be successful. For the capacity achieving distribution R_C, the probability that v will not interrupt a transmission from u is $\frac{x-1}{x}$. Given that a packet from u finds v idle, the probability that v stays idle goes to 1 as x goes to infinity. But, as x grows, the expected length of an idle period becomes large, which can lead to undesirable latency characteristics. Low values of x result in shorter idle periods. The link throughput has the following formulation as a function of x:

$$T(x) = \lambda_C \cdot P_{idle,C} \cdot P_{Y_C} \tag{11}$$
$$= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{x-1}{x} \tag{12}$$

The tradeoff of system throughput versus idle time x is shown in Figure 2. We observe that most of the throughput gain is achieved at relatively small values of x. For $x = 5$, 80% of the network capacity is achieved, and at $x = 10$, 90% of network capacity is achieved. As described earlier, with $x \to \infty$, the network capacity is reached asymptotically.

For asymptotic analysis, we assumed that a user always has a packet to send to each of its neighbors during some transmit period. We now examine delay characteristics for a network with stochastic arrivals. At some particular node, we

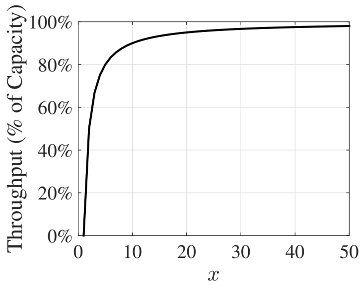


Fig. 2: Percent of the network capacity achieved vs. x

model the arrival rate of packets destined to neighbor i as Poisson distributed with parameter γ_i. The capacity achieving distribution R_C has an expected service rate of $\mathbb{E}[R_C] = 1$ and second moment of service rate of $\mathbb{E}[R_C^2] = x$. The link utilization factor to user i is $\rho_i = \gamma_i \mathbb{E}[R_C] = \gamma_i$.

The average wait time $\bar{W}_i$ of a packet in queue i in a system with a Poisson distribution for arrivals and a general distribution for service time (known as an $M/G/1$ queue) is given by the Pollaczek-Khinchin formula [15]:

$$\bar{W}_i = \frac{\gamma_i \mathbb{E}[R_C^2]}{2(1-\rho_i)} = \frac{\gamma_i}{1-\gamma_i}\frac{\mathbb{E}[R_C^2]}{2} = \frac{\rho_i}{1-\rho_i}\frac{x}{2} \tag{13}$$

We note that the classic $M/M/1$ queuing result has an average wait time of $\frac{\rho}{1-\rho}$ for a packet in a queue [15]. For multi-beam networks, the capacity achieving distribution behaves similarly to an $M/M/1$ queue. In particular, operating the network near capacity (i.e., $\rho_i \to 1$) causes average packet wait times to go to infinity. If we consider a system operating at $\rho_i = \frac{1}{2}$ and $x = 5$, we find that a packet waits in the queue on average 2.5 units of time before being transmitted, which is half the time of a full idle period (5 in this case). On average, a packet arriving to a node will find that node in transmit mode 50% of the time, and the packet will encounter little to no wait time in the queue. When a packet does arrive at an idle node, on average, the packet will arrive in the middle of the idle period, and it will not have to wait the entire idle time before the next transmission period begins.

C. Multi-Beam Random Access under Realistic Beamwidths

We now remove the assumption that beamwidths are arbitrarily narrow. To analyze the effect that wider beamwidths cause on network performance, we use the flat-top beam model as described in Section III. We assume that a two users point the center of their transmit and receive beams at one another. Hence, at the receiver, if two transmissions are within $\frac{1}{2}\theta$ degrees of one another, then the two beams interfere and neither message can be received. Similarly, at the transmitter, beams cannot be formed to users that are fewer than $\frac{1}{2}\theta$ degrees apart of one another. This reception and transmission model was demonstrated in Figure 1. Our goal is to understand the effect that beamwidth has on the achievable network throughput.

To simplify analysis, we use the following model. We consider the network from the perspective of a single user v. Nodes are uniformly distributed according to a two dimensional spatial Poisson process with an average of n users per square unit area. We assume v has a maximum range of $\pi^{-\frac{1}{2}}$, and that any user within this range is considered a neighbor. Hence, all neighbors of v exist in an area of one square unit, resulting in an expected value of n neighbors. We assume that all users in the network have the same beamwidth. All users in the network operate independently and in an uncoordinated fashion. Hence, the behavior of any node is independent of the behavior of any other node in the network. We first consider analysis at the receiver, and then we examine the transmitter.

1) Receiver Analysis: We analyze the rate of successful transmissions from u to v. The link throughput is written as $T = \lambda P_{recv}$, where λ is the rate of transmission from u to v and P_{recv} is the probability that any packet is received successfully at v. Unlike the link throughput for the asymptotic analysis, P_{recv} is not simply the probability that the receiver is idle at the time of packet arrival and that it remains idle for the duration of one packet; we now consider interference from other users that are within the same beamwidth of u. Using the capacity achieving distribution R_C, users are transmitting half of the time and receiving half of the time. At the time of packet arrival from u, v must not be actively receiving packets from users that are within $\frac{1}{2}\theta$ degrees in either direction of u, for a total of θ. Using distribution R_C, we label this probability $P^{u,\theta}_{idle_C}$. The number of users that are within $\frac{1}{2}\theta$ degrees in either direction of u is labeled as random variable Z, where Z is Poisson distributed with parameter $n\frac{\theta}{360}$. We label the event that v is not receiving a packet from any user in Z at the time a packet arrives from u as Z^u_{idle}. We now derive $P^{u,\theta}_{idle_C}$.

$$\begin{aligned} P^{u,\theta}_{idle_C} &= \mathbf{P}(Z^u_{idle}) && (14)\\ &= \sum_{z=0}^{\infty} \mathbf{P}(Z^u_{idle}|Z=z)\cdot \mathbf{P}(Z=z) && (15)\\ &= \sum_{z=0}^{\infty}\left(\frac{1}{2}\right)^z \cdot \frac{(n\theta/360)^z}{z!}\mathrm{e}^{-n\theta/360} && (16)\\ &= \sum_{z=0}^{\infty}\frac{(n\theta/720)^z}{z!}\mathrm{e}^{-2n\theta/720} && (17)\\ &= \mathrm{e}^{-n\theta/720}\sum_{z=0}^{\infty}\frac{(n\theta/720)^z}{z!}\mathrm{e}^{-n\theta/720} && (18)\\ &= \mathrm{e}^{-n\theta/720} && (19)\end{aligned}$$

We note that the summation in (18) is across the entire probability mass function of a Poisson distribution with parameter $n\frac{\theta}{720}$, which sums to 1.

Next, given that v begins successfully receiving a packet from u, all users within $\frac{1}{2}\theta$ degrees of u must remain idle for at least one packet in length; we label this probability $P^{u,\theta}_{Y}$. Each node has a probability of not transmitting for at least one packet in time starting at t of $\mathbf{P}(Y(t) > 1)$. Using the capacity achieving distribution R_C, $\mathbf{P}(Y_C(t) > 1) = \frac{x-1}{x}$, which goes to 1 as $x \to \infty$. Using similar analysis as was used to derive $P^{u,\theta}_{idle}$, we find the following:

$$P^{u,\theta}_{Y_C} = e^{-n\theta/720x} \tag{20}$$

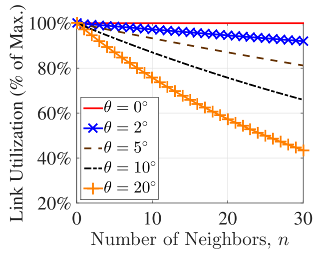


Fig. 3: Percent of maximum link utilization achieved for different beamwidths with respect to the average number of neighbors n

As x goes to infinity, the probability that a transmission from any neighboring user will interrupt the ongoing packet transmission from u at v goes to zero. We can now calculate the per link throughput from u to v for a system with beamwidth θ. The throughput formulation is similar to that done for the asymptotic analysis, but now includes the addition of interference from users that are within $\frac{1}{2}\theta$ degrees of u.

$$T_C^\theta = T_C \cdot P_{idle_C}^{u,\theta} \cdot P_{Y_C}^{u,\theta} \tag{21}$$

$$= \lambda \cdot P_{idle_C} \cdot P_{Y_C} \cdot P_{idle}^{u,\theta} \cdot P_{Y_C}^{u,\theta} \tag{22}$$

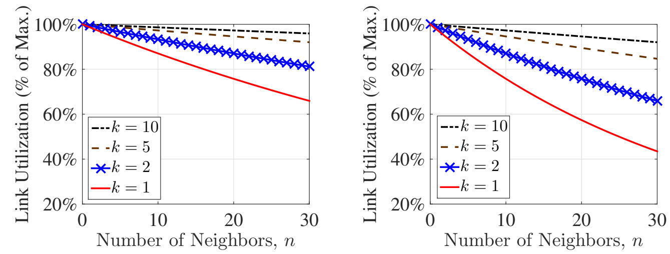
$$= \frac{1}{4} \cdot \frac{x-1}{x} \cdot \mathrm{e}^{-n\theta/720} \cdot \mathrm{e}^{-n\theta/720x} \tag{23}$$

The final result for link throughput T_C^θ matches intuition. As beamwidth θ goes to zero, the impact of any interfering neighbor becomes negligible. As x goes to infinity, the probability that a transmission from u to v will be interrupted by the start of a transmission from either v or from some interfering neighbor goes to zero.

In Figure 3, results are shown for different values of θ when $x = \infty$ with respect to the number of neighbors a user has. For narrow beamwidths, there is little loss with an increased number of neighbors. For 2° beamwidth, over 30 neighbors can be supported with only a 10% loss in maximum link utilization. A 5° beamwidth is able to support 90% of link capacity with 15 neighbors, and 80% with 30 neighbors. At wider beamwidths, not surprisingly, fewer users can be supported. At 20°, 8 neighbors can still be supported at 80% of link capacity, but with 20 neighbors, capacity drops to 50%.

We now look to improve performance for systems with wider beamwidths. Up to this point, we assumed a single-channel system where any two transmissions towards some receiver that are within $\frac{1}{2}\theta$ degrees of one another will interfere. To improve the reception probability of our uncoordinated MAC, we propose the following system modification: increase the number of channels from 1 to k. If two transmissions are within a beamwidth of one another at a receiver, but are on two different channels, then we assume that they do not interfere. We still maintain half-duplex constraints at the receiver.

We implement the following simple policy for a multi-channel system: at time of transmission, a user randomly selects one of the k available channels for each outgoing packet. We label the probability that at a particular time, v is not receiving any packets from any users on channel k as $P_{idle_C}^{u,\theta,k}$. Given that v begins successfully receiving a packet on channel k from node u, we label the probability that no packet



(a) Beamwidth of 10°

(b) Beamwidth of 20°

Fig. 4: Percent of maximum link utilization achieved for beamwidths of 10° and 20° with respect to the number of channels k and number of neighbors n

arrives at v on channel k from any nodes within $\frac{1}{2}\theta$ of u as $P_{Y_C}^{u,\theta,k}$. We leave out the full derivation of $P_{idle_C}^{u,\theta,k}$ and $P_{Y_C}^{u,\theta,k}$ due to space constraints.

$$P_{idle_C}^{u,\theta,k} = \mathrm{e}^{-n\theta/720k} \tag{24}$$

$$P_{Y_C}^{u,\theta,k} = \mathrm{e}^{-n\theta/720kx} \tag{25}$$

In Figure 4, results are shown for beamwidths of 10° and 20° and different values of k when $x = \infty$. Moving from one to two channels offers significant improvements. For 10° beamwidth with 15 neighbors, maximum link utilization goes from 80% to 90%. At 30 neighbors, two channels allows 80% of maximum utilization to still be obtained with 30 neighbors. For 20° beamwidth with 15 neighbors, link utilization goes from 65% to 80% of the maximum. Going to five channels offers significantly increased improvements, and at ten frequency channels, almost the entire maximum link utilization is achievable for both 10° and 20° beamwidths operating with a large number of neighbors.

2) Transmitter Analysis: We next consider transmissions from user v to its neighbors. When beamwidth goes to zero, v can transmit to all of its neighbors without having any of those transmissions interfere. But, when beamwidths grow large, beams cannot be formed to users that are fewer than $\frac{1}{2}\theta$ degrees apart of one another. To fairly serve users, we propose two greedy approaches for deciding to whom to transmit: (1) Random Shuffle and (2) Greedy Max-Weight.

For the Random Shuffle approach, in each transmit period, randomly permute the users and pick users starting from the beginning of the list. Consider two users a and b, ordered 1 and 2, respectively. If a transmit beam to user b interferes with a beam to user a, a beam will be formed to a and not to b. Every transmit period, there is a 50% chance that user a will be ahead of user b. This provides a fair allocation of resources across all of the potential beams.

For the Greedy Max-Weight approach, each neighbor will have a cost assigned to its link according the size of that link's queue backlog (similar to how costs were assigned to link in Section V-A). For Greedy Max-Weight, sort users according their cost, and in a similar fashion as Random Shuffle, pick users from the beginning of the list. If two transmit beams interfere, the one that is higher on the list will be selected. As a link's queue grows in size, it will move higher on the list, and that link will eventually be selected.

D. Discovery for Adaptive Multi-Beam Directional Networks

We previously assumed that the location of all neighbors are known, allowing users to properly form transmit beams to the correct location and at the correct power level. While this may be the case in static or slow-moving networks, in many networks, the set of possible neighbors and their respective locations change and need to be continuously discovered.

In directional networks without digital beamforming capabilities, receive beams must be formed before a transmission begins to successfully receive a packet. This requires precise time synchronization and designated discovery time slots. The discovery problem is made even more complex when only a small number of beams can be activated simultaneously. Users search for neighbors in all possible directions, and a complex coordination algorithm is required to ensure that two users will be guaranteed to have a transmit and receive beam point at one another at some point in time [28]. To avoid this complexity, some papers have suggested that for discovery, receivers operate in omni-directional mode [20]. While this does reduce some of challenges associated with directional discovery, it eliminates many of the benefits of directionality. An omni-directional receive mode for discovery results in reduced antenna gain, higher interference between packets at the receiver, and for the case of military networks, increased risk of being jammed.

With digital beamforming, the discovery process is greatly simplified. Transmitters can use any transmit period to send discovery messages, and users can send discovery messages across a subset of beams and data packets across others. On the receive side, receivers can listen across all beams in all directions simultaneously, and any user in idle mode can receive multiple simultaneous discovery messages from different users without the need for any designated discovery periods. Once a user receives a discovery message, it learns the location of the transmitter, and it can send a return message to establish a link between the two.

VI. Conclusion

In this paper, we presented a multi-beam uncoordinated random access MAC (MB-URAM) for emerging systems capable of adaptive digital beamforming. MB-URAM asymptotically achieves the network capacity upper bound as beamwidth goes to zero, the number of users grows large, and latency requirements are relaxed.

We then considered practical considerations on the performance of MB-URAM, including power constraints, latency, beamwidth, and neighbor discovery. Analysis was performed showing that MB-URAM still performs well even when realistic constraints are imposed. Numerous algorithms were proposed to help improve the performance of MB-URAM under these practical constraints.

Future areas of study include analyzing the performance of MB-URAM using higher fidelity physical layer models, proposing appropriate channel coding techniques, and performing analysis and evaluation of MB-URAM for multi-hop networks.

References

[1] J. Litva and T. K. Lo, *Digital beamforming in wireless communications*. Artech House, Inc., 1996.

[2] B. D. Van Veen and K. M. Buckley, "Beamforming: A versatile approach to spatial filtering," *IEEE assp magazine*, vol. 5, no. 2, pp. 4–24, 1988.

[3] G. Krieger, N. Gebert, and A. Moreira, "Multidimensional waveform encoding: A new digital beamforming technique for synthetic aperture radar remote sensing," *Geoscience and Remote Sensing, IEEE Transactions on*, vol. 46, no. 1, 2008.

[4] A. E. Sayers, W. M. Dorsey, K. W. O'Haver, J. Valenzi *et al.*, "Planar near-field measurement of digital phased arrays using near-field scan plane reconstruction," *Antennas and Propagation, IEEE Transactions on*, vol. 60, no. 6, 2012.

[5] A. S. Prasad, S. Vasudevan, R. Selvalakshmi, K. S. Ram, G. Subhashini, S. Sujitha, and B. S. Narayanan, "Analysis of adaptive algorithms for digital beamforming in smart antennas," in *Recent Trends in Information Technology (ICRTIT), 2011 International Conference on*. IEEE, 2011, pp. 64–68.

[6] M. Ascione, G. Bernardi, A. Buonanno, M. D'Urso, M. Felaco, M. G. Labate, G. Prisco, and P. Vinetti, "Simultaneous beams in large phased radar arrays," in *Phased Array Systems & Technology, IEEE International Symposium on*. IEEE, 2013, pp. 616–616.

[7] M. Di Filippo, L. Lucci, D. Marabissi, and S. Selleri, "Design of a smart antenna for mobile ad hoc network applications," *International Journal of Antennas and Propagation*, 2015.

[8] L. C. Godara, "Application of antenna arrays to mobile communications. ii. beamforming and direction-of-arrival considerations," *Proceedings of the IEEE*, vol. 85, no. 8, pp. 1195–1245, 1997.

[9] I. Bekmezci, O. K. Sahingoz, and Ş. Temel, "Flying ad-hoc networks (fanets): A survey," *Ad Hoc Networks*, vol. 11, no. 3, pp. 1254–1270, 2013.

[10] J. Lai, "Broadband wireless communication systems provided by commercial airlines," Sep. 4 2001, US Patent 6,285,878.

[11] B.-N. Cheng and S. Moore, "A comparison of manet routing protocols on airborne tactical networks," in *MILCOM*. IEEE, 2012, pp. 1–6.

[12] I. Jacobs, E. Hoversten, and C. Moore, "Trends in military satellite communication networks," *American Institute of Aeronautics and Astronautics, Inc*, pp. 762–765, 1980.

[13] T. Kim, J. Park, J.-Y. Seol, S. Jeong, J. Cho, and W. Roh, "Tens of gbps support with mmwave beamforming systems for next generation communications," in *GLOBECOM*. IEEE, 2013, pp. 3685–3690.

[14] M. K. Tsatsanis, R. Zhang, and S. Banerjee, "Network-assisted diversity for random access wireless networks," *Signal Processing, IEEE Transactions on*, vol. 48, no. 3, pp. 702–711, 2000.

[15] D. P. Bertsekas, R. G. Gallager, and P. Humblet, *Data networks*. Prentice-Hall International New Jersey, 1992, vol. 2.

[16] L. Bao and J. Garcia-Luna-Aceves, "Transmission scheduling in ad hoc networks with directional antennas," in *Proceedings of the 8th annual international conference on Mobile computing and networking*. ACM, 2002, pp. 48–58.

[17] K.-W. Chin, S. Soh, and C. Meng, "Novel scheduling algorithms for concurrent transmit/receive wireless mesh networks," *Computer Networks*, vol. 56, no. 4, pp. 1200–1214, 2012.

[18] J.-L. Hsu and I. Rubin, "Directional random access scheme for mobile ad hoc networking using beamforming antennas," *Ad Hoc Networks*, vol. 6, no. 1, pp. 127–153, 2008.

[19] V. Jain, A. Gupta, and D. P. Agrawal, "On-demand medium access in multihop wireless networks with multiple beam smart antennas," *Parallel and Distributed Systems, IEEE Transactions on*, vol. 19, no. 4, pp. 489–502, 2008.

[20] O. Bazan and M. Jaseemuddin, "A survey on mac protocols for wireless adhoc networks with beamforming antennas," *Communications Surveys & Tutorials, IEEE*, vol. 14, no. 2, pp. 216–239, 2012.

[21] S. Singh, R. Mudumbai, and U. Madhow, "Interference analysis for highly directional 60-ghz mesh networks: The case for rethinking medium access control," *IEEE/ACM Transactions on Networking (TON)*, vol. 19, no. 5, 2011.

[22] S. Venkatesan, A. Lozano, and R. Valenzuela, "Network mimo: Overcoming intercell interference in indoor wireless systems," in *Signals, Systems and Computers. ACSSC. Conference Record of the Forty-First Asilomar Conference on*. IEEE, 2007, pp. 83–87.

[23] R. G. Gallager, *Discrete stochastic processes*. Springer Science & Business Media, 2012, vol. 321.

[24] D. J. MacKay, "Fountain codes," *IEE Proceedings-Communications*, vol. 152, no. 6, pp. 1062–1068, 2005.

[25] M. J. Neely, E. Modiano, and C. E. Rohrs, "Power allocation and routing in multibeam satellites with time-varying channels," *IEEE/ACM Transactions on Networking (TON)*, vol. 11, no. 1, pp. 138–152, 2003.

[26] M. Magazine and O. Oguz, "A fully polynomial approximation algorithm for the 0–1 knapsack problem," *European Journal of Operational Research*, vol. 8, no. 3, pp. 270–273, 1981.

[27] P. He, L. Zhao, S. Zhou, and Z. Niu, "Water-filling: A geometric approach and its application to solve generalized radio resource allocation problems," *Wireless Communications, IEEE Transactions on*, vol. 12, no. 7, pp. 3637–3647, 2013.

[28] G. Jakllari, W. Luo, and S. V. Krishnamurthy, "An integrated neighbor discovery and mac protocol for ad hoc networks using directional antennas," *Wireless Communications, IEEE Transactions on*, vol. 6, no. 3, pp. 1114–1024, 2007.